The Isgur-Wise function on the lattice

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UKQCD collaboration
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The h_+, h_{A_1} form factors for the semi-leptonic $B \to D$ and $B \to D^*$ decays are evaluated in quenched lattice QCD with $\beta = 6.2$. The action and the operators are fully $\mathcal{O}(a)$ non-perturbatively improved. The Isgur-Wise function is evaluated and fitted to extract its slope; the latter is found to be $\rho^2 = 1.1(2)(3)$ from the $B \to D^*$ decay and $\rho^2 = 1.0(2)(3)$ from the $B \to D$ decay. The form factors ratios R_1, R_2 are evaluated and found to be in agreement with experimental determinations.

1. Introduction

The $B \to D$ and $B \to D^*$ semi-leptonic decays have a considerable phenomenological interest, since they can be studied to determine the modulus of CKM matrix element $V_{\rm cb}$. Furthermore, the presence of non-perturbative physics in these decays makes a lattice study particularly appealing. The predictions of the Heavy Quark Effective Theory (HQET) can be used to constrain the form of the matrix elements that describe semi-leptonic decays of heavy-light mesons. In particular, the relevant matrix elements are expressed in terms of a set of form factors, that contain the non-perturbative physics of the decay:

$$\frac{\langle D(v')|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_D}} = (v+v')^\mu h_+(\omega) + (v-v')^\mu h_-(\omega) \ , \tag{1}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = i\epsilon^{\mu\nu\rho\sigma}\epsilon^*_\nu v'_\rho v_\sigma h_V(\omega) \ , \tag{2}$$

$$\frac{\langle D^*(v',\epsilon)|\bar{c}\gamma^\mu\gamma^5 b|B(v)\rangle}{\sqrt{m_B m_{D^*}}} = (\omega+1)\epsilon^{*\mu}h_{A_1}(\omega) + -[h_{A_2}(\omega)v^\mu + h_{A_3}(\omega)v'^\mu](\epsilon^*\cdot v) \tag{3}$$

where v, v' are the velocities of the initial and the final meson respectively, and $\omega = v\cdot v'$; ϵ is the polarisation vector of the D^* meson. In the heavy quark limit, the six form factors become related to a universal function known as the Isgur-Wise function, $\xi(\omega)$. However, there are two kinds of corrections to the heavy quark symmetry that one has to take into account: perturbative QCD corrections and symmetry breaking corrections proportional to inverse powers of the heavy quark mass. In terms of these corrections, the relations between the form factors and $\xi(\omega)$ are [1]:

$$h_j(\omega) = [\alpha_j + \beta_j(m_b, m_c;\omega) + \gamma_j(m_b, m_c;\omega) + ...]\xi(\omega) \ , \tag{4}$$

The α_j terms are constants that fix the behaviour of the form factors in absence of corrections ($\alpha_+ = \alpha_V = \alpha_1 = \alpha_3 = 1$; $\alpha_- = \alpha_2 = 0$). The β_j and γ_j functions account for radiative corrections and power corrections of $\mathcal{O}(1/m_{b,c})$ respectively. The radiative corrections are calculable in perturbation theory, while the power corrections are non-perturbative in nature [2]. At zero recoil ($v = v', \omega = 1$), however, Luke's theorem [3] guarantees that both γ_+ and γ_{A_1} are of order $\mathcal{O}(1/m^2_{b,c})$. At small recoil, the Isgur-Wise function is modeled by:

$$\xi(\omega) = 1 - \rho^2(\omega-1) + \mathcal{O}((\omega-1)^2) \ , \tag{5}$$

where ρ^2 is called the slope parameter and $\xi(1) = 1$ because of current conservation. Alternative parametrisations of $\xi(\omega)$ have been proposed, which start to differ from (5) at $\mathcal{O}((\omega-1)^2)$ [4,5].

2. Simulation details

The calculations presented in this work were performed using a non-perturbatively $\mathcal{O}(a)$ improved action (SW, $c_{\rm SW} = 1.614$), within the quenched approximation, at $\beta = 6.2$, on a set of 216 configurations, with a volume of $24^3 \times 48$. The improvement coefficients for the current operators were taken from the work of Bhattacharya *et al.* [6]. The inverse lattice spacing, set using the Sommer scale r_0 [7], was $a^{-1} \simeq 2.9$ (GeV). The matrix elements have been extracted for eight combinations of the heavy quark masses ($m_Q \simeq m_{\rm charm}$) and two values of the mass of light (passive) quark ($m_q \simeq m_{\rm strange}$). Since, at fixed ω, the residual dependence of $\xi(\omega)$ on the heavy quark mass was statistically negligible, no extrapolations were necessary.

3. Extraction of the Isgur-Wise function

The Isgur-Wise function has been extracted independently from the h_+ and h_{A_1} form factors, that at $\omega = 1$ are protected from $\mathcal{O}(1/m_{\rm b,c})$ power corrections by Luke's theorem:

$$\xi(\omega) \simeq \frac{h_+(\omega)}{1+\beta_+(\omega)} , \qquad (6)$$

$$\xi(\omega) \simeq \frac{h_{A_1}(\omega)}{1+\beta_{A_1}(\omega)} \qquad (7)$$

The power corrections have been neglected, as they have been estimated and found to be consistent with zero in the range $1.0 \leq \omega \leq 1.2$. It is not possible to extract the Isgur-Wise function from the form factors for which $\alpha_j = 0$ as they are just a collection of QCD corrections; similarly, the h_V form factor could not be used because its power correction has been found large. The Isgur-Wise function has been fitted to the linear model (5). Unconstrained fits have also been performed, i.e. relaxing the condition $\xi(1) = 1$. In both decays, $\xi(1)$ was found to be consistent with one. Results are summarized in Table (1) and plotted in Figure (1). The data from the two decays are in excellent agreement with each other and lie consistently on the same line.

Table 1
Results of the linear fits the Isgur-Wise function, with and without the $\xi(1) = 1$ constraint. Quoted errors are statistical.

	$B \to D$		$B \to D^*$	
	con. fit	un. fit	con. fit	un. fit
$\xi(1)$	$\equiv 1$	0.99(2)	$\equiv 1$	1.0(3)
ρ^2	1.0(2)	0.9(3)	1.1(2)	1.1(2)

4. Ratios of form factors

The two following ratios of form factors have also been calculated:

$$R_1(\omega) = \frac{h_V(\omega)}{h_{A_1}(\omega)} , \qquad (8)$$

$$R_2(\omega) = \frac{h_{A_3}(\omega) + \frac{M_{\rm D*}}{M_{\rm B}} h_{A_2}(\omega)}{h_{A_1}(\omega)} \qquad (9)$$

These two ratios would be equal to one in the absence of symmetry breaking corrections. Figures (2) and (3) show that this work's determinations are in agreement with the experimental determinations by the CLEO collaboration [8].

5. Systematic uncertainties

The systematic uncertainty on the slope of the Isgur-Wise function has been estimated considering the effect of the choice of the scale-fixing quantity, the residual light quark mass dependence, and the effect of using different non-zero momentum kinematic channels. Three alternative models of the Isgur-Wise function have been considered, and the variation in the extracted slope parameter has been taken as a systematic uncertainty.

6. Summary and conclusions

The slope of the Isgur-Wise function has been extracted from a lattice study of the $B \to D$ and $B \to D^*$ semi-leptonic decays, using a $\mathcal{O}(a)$ non-perturbatively improved action at $\beta = 6.2$. The results from the two decays have been found in good agreement: $\rho^2 = 1.1(2)(3)$ from $B \to D^*$

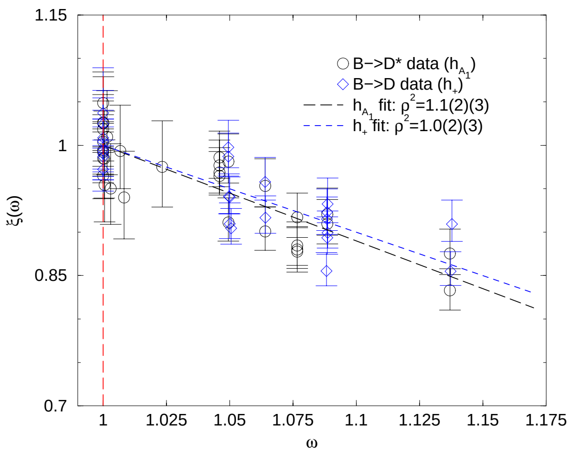

Figure 1. The Isgur-Wise function from the $B \to D$ and $B \to D^*$ semi-leptonic decays.

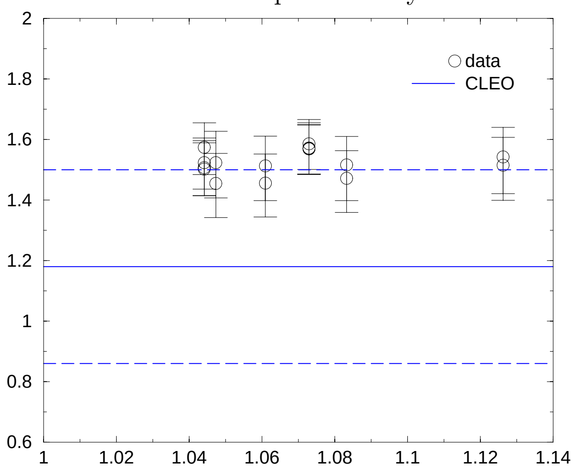

Figure 2. The R_1 ratio. The range of the experimental determination is shown.

and $\rho^2 = 1.0(2)(3)$ from $B \to D$. A previous analysis of the $B \to D$ semi-leptonic decay on the data-set used in this work by the UKQCD collaboration [9], that employed a perturbative estimate of the vector current improvement coefficient c_V, gave a result of $\rho^2 = 1.1(2)$. The ratios of form factors R_1, R_2 have been calculated and found in agreement with experimental data.

7. Acknowledgements

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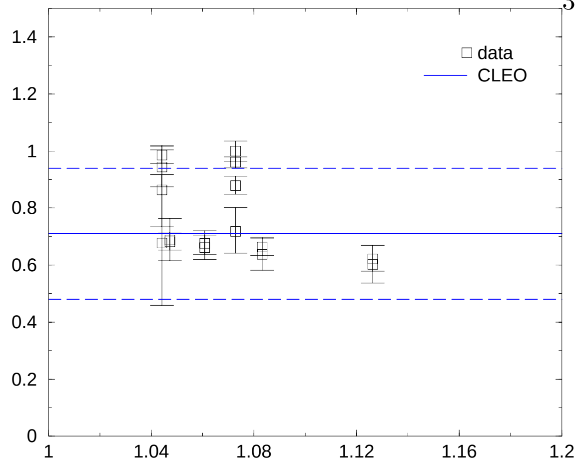

Figure 3. The R_2 ratio. The range of the experimental determination is shown.

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